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Plasmoids can be described as a group of electrons and ions that propagate as a unit through vacuum. In their present conception they consist of an expanding halo region, carrying return current and bearing a slight charge imbalance, and an equilibrium core region that propagates like a pinched charged particle beam. This model was first suggested by T. Lockner of Sandia. Since they are a type of charged particle beam, they can be generated with higher efficiency and deposit their energy over a deeper distance in matter than laser beams. This makes plasmoids attractive for defense applications; however, the structure, formation, and propagation of plasmoids in a space environment are not known at present. Theory and one-dimensional simulations aimed at finding the core structure of plasmoids by using variational principles have been successful. This report documents the progress made in this first phase of research. A proposed second phase of research will use similar techniques to model the three-dimensional formation and propagation of plasmoids in a space environment.

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FINAL REPORT

THEORY AND SIMULATION OF RELAXED PLASMOIDS

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INTRODUCTION

1.1   PLASMOIDS

Plasmoids can be described as a group of electrons and ions that propagate as a unit through vacuum. In their present conception they consist of an expanding halo region, carrying return current and bearing a slight charge imbalance, and an equilibrium core region that propagates like a pinched charged particle beam. This model was first suggested by T. Lockner of Sandia. These parts are illustrated in Figure 1. Since they are a type of charged particle beam, they can be generated with higher efficiency and deposit their energy over a deeper distance in matter than laser beams. This makes plasmoids attractive for defense applications; however, the structure, formation, and propagation of plasmoids in a space environment are not known at present. Theory and one-dimensional simulations aimed at finding the core structure of plasmoids by using variational principles have been successful. This report documents the progress made in this first phase of research. A proposed second phase of research will use similar techniques to model the three-dimensional formation and propagation of plasmoids in a space environment.

1.2   PROGRESS IN PHASE-ONE RESEARCH

The first phase of research was aimed at understanding the core equilibrium of a plasmoid as being the result of a relaxation process. In this model the plasmoid was a minimum energy structure, and internal structure could be found by variational calculus. This model was suggested by the success of such methods in magnetic fusion work (Reference 1).

Progress has been made in the theory of plasmoid core equilibria of three types (using cylindrical coordinates with z being the equilibrium axis):

(1) nonrotational equilibria,

(2) rotational equilibria with no axial magnetization
($V_\theta \neq 0$ $J_\theta = 0$), and

(3) rotational, axially magnetized equilibria ($V_\theta \neq 0$ $J_\theta \neq 0$
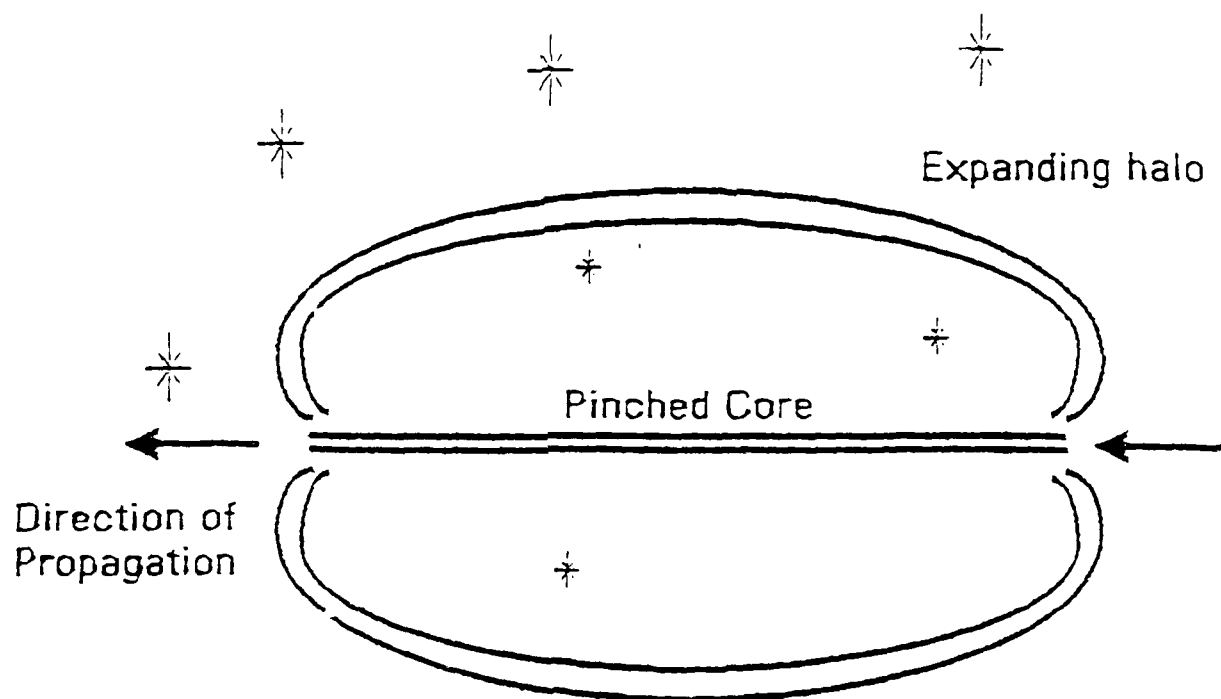leading to a B_z field).

Figure 1. A plasmoid model that was first proposed by Tom Lockner of Sandia National Laboratories.

The variational method, called the method of Lagrange multipliers (Reference 2), was applied originally in one Lorentz frame. This frame of reference, called the z-pinch frame, is a frame where the core equilibrium is charge neutral and electrons and ions move past each other, so their currents add. The plasmoid core equilibrium can, therefore, be understood as a Lorentz transformed z pinch. However, the variational principal originally was formulated in the z-pinch frame only and assumed similar density profiles for electrons and ions. A generalized variational principle, valid in all Lorentz frames and requiring similar profiles for both electrons and ions in all frames, has now been formulated. The principal of Lorentz invariance strongly indicated the form of the variational problem and the Lorentz invariant variational principal was found to predict the three types of equilibria mentioned earlier.

Simulations of the three types of equilibria discussed were performed assuming rotational and axial symmetry, that is, effectively one-dimensional. The simulations consisted of equilibria initialized in steady-state and nonequilibrium configurations that were allowed to relax to equilibrium. The simulations demonstrated that the equilibria predicted by variational theory did exist and were unique end states of the relaxation process. Simulations also demonstrated that relaxation to final profiles was slower than the expected ion transit time timescale, and instead proceeds according to a Bohm diffusion timescale.

1.3   PLASMOID EQUILIBRIA AND Z-PINCH FRAME

Before proceeding to the body of this report it is useful to briefly consider the equations of plasmoid equilibria. A simple method to generate plasmoid equilibria, which also aids in understanding them, is to use the two dimensional Virial Theorem (Reference 3). This theorem gives necessary conditions for the existence of an infinitely long rotationally symmetric, pinched equilibrium. This condition is:

$$I^2/c^2 - Q^2 = 2NkT \quad , \tag{1.1}$$

where I is the total current in the equilibrium, Q is the total charge per unit length, 2NkT is the total thermal energy per unit length and c is the speed of light. This condition can be understood as a generalization of the Bennett pinch condition (Reference 4).

The quantities on both sides of Equation (1.1) can be shown to be Lorentz invariants. Thus, a radial equilibrium in one frame can be shown to be a radial equilibrium in all other Lorentz frames. This property of Lorentz invariance was used to create plasmoid beam equilibria in one frame, a frame in which both electrons and ions moved together at nearly the speed of light, by setting up a charge neutral (per unit length) z-pinch in a frame where electrons and ions moved slowly in opposite directions at equal and opposite velocities β_0 where $\beta_0 \ll 1$. In this z-pinch frame Equation (1.1) becomes,

$$I^2/c^2 = I'^2/c^2 - Q'^2 = 2N'kT' = 2NkT,$$
(1.2)

where the primed quantities are transformed into the z-pinch frame. The plasmoid equilibrium was then merely a rapidly moving z-pinch. The electron and ion densities for equilibrium are then related by the ratio

$$n_i/n_e = \gamma_i/\gamma_e$$
(1.3)

where n_i and n_e are the electron and ion densities respectively and where γ_i and γ_e are the relativistic gamma factors for the ions and electrons respectively, $\gamma = (1 - \beta^2)^{-1/2}$ where $\beta = v/c$.

One type of equilibria that was found was that of a Bennett profile equilibrium with electrons and ions having similar profiles. The profiles were solutions to the set of coupled equations:

$$-\frac{dn_e}{dx} kT_e = \frac{n_e \pi e^2}{x} \int_0^x n_e [(\beta_e^2 - 1) - \frac{\gamma_i}{\gamma_e}(\beta_i \beta_e - 1)] \, dx$$
(1.4a)

$$-\frac{dn_i}{dx} kT_e = \frac{n_i \pi e^2}{x} \int_0^x n_i [(\beta_i^2 - 1) - \frac{\gamma_i}{\gamma_i}(\beta_i \beta_e - 1)] \, dx$$
(1.4b)

where kT_e and kT_i are the transverse temperatures of electrons and ions, $x = r^2$, and where β_e and β_i are the normalized electron and ion velocities, respectively. The solutions are found to be

$$n_\alpha = n_{\alpha o}/(1 - (x/a)^2)^2 \tag{1.5}$$

where $n_{\alpha o}$ is the density on axis for either specie and where the Bennett radius, a, is defined as:

$$a^2 = \frac{2kT_\alpha \, \gamma_\alpha^2}{\pi e^2 (\gamma_i \gamma_e (1 - \beta_e \beta_i) - 1) n_{\alpha o}} \tag{1.6}$$

It was shown further that the quantity $\gamma_i \gamma_e (1 - \beta_i \beta_e)$ could be expressed as $\cosh(\varepsilon_i - \varepsilon_e)$ where ε_i and ε_e are equal to $\tanh^{-1} \beta_i$ and $\tanh^{-1} \beta_e$ respectively, and that this quantity was itself invariant between frames, as was $\gamma_e kT_e/n_i$, so that the Bennett radius in all frames was equal to the value it assumed in the z-pinch frame:

$$a^{2\prime} = \frac{kT'}{\pi e^2 n'_o \beta'^2_o} \tag{1.7}$$

where n'_o and kT'_o are the common densities and transverse temperatures of the electrons and ions in the z-pinch frame. The quantity $2\beta_o^2$ becomes equal to $\cosh(\varepsilon_i - \varepsilon_e)$ in the limit that β_o, the drift velocity of the electrons and ions in the z-pinch frame, is small. The slow drift speed in the z-pinch frame is one of the initial simplifying assumptions of the model.

It has been observed in simulations of plasmoid equilibria of general initial profiles that electron and ion profiles evolve very quickly to the same shape if their initial profiles are dissimilar. Also it has been observed that once the profiles of electrons and ions become similar, there is a slower evolution to a Bennett profile. This is true regardless of the initial profiles for the plasmoids.

It will be one purpose of this report to explain why similar Bennett profiles for electrons and ions should be expected for any nonrotating plasmoid after many ion betatron periods. The reasons for this expectation, it will be

shown, involve the tendency for systems to seek their lowest accessible energy state and also their tendency to move to a state of maximum entropy. Possible final states for beams with initial cannonical angular momentum are also of interest and will be discussed.

The method chosen for investigating these equilibria issues will be a variational approach. Variational principals have been shown to be powerful techniques for predicting the end conditions of very complicated systems. They have found increasing use in magnetic confinement fusion theory to predict the results of experiments (Reference 1).

Because of its compactness we will use fluid theory to describe beam equilibria and their evolution. The same results could be obtained through Vlasov theory, but not without the loss of simplicity and easy understanding.

1.4   BASIC ASSUMPTIONS OF RELAXATION MODELS

When a system evolves to maximize or minimize some global quantity the system is said to "relax". The relaxation of systems to minimum energy states or maximum entropy is observed often in everyday life. The movement of water in a sink to assume a common lowest level when more water is added is an example of energy minimization. The diffusion of heat in a frying pan until its surface is a uniform temperature can be thought of as a maximization of entropy. It seems reasonable to expect similar behavior in non-everyday systems; however, in everyday systems the mechanisms for relaxation and dissipation are clearly understood. The production and subsequent phase-mixing and damping of water waves allows water to reach a common level and the process of heat conduction by microscopic processes causes the frying pan to relax to a uniform temperature. Therefore, when we consider the relaxation of intense particle beam equilibria we must consider what process or processes will lead to dissipation and increases in entropy. In beam equilibria the particles will move in a complicated system of interweaving orbits. Actual close approaches by individual particles to one another so as to strongly perturb each others motion will be rare; however, strong oscillations of local space-charge density, caused

by the collective interaction of particles, can strongly perturb the motion of particles, both in groups of particles passing through each other (a two stream instability) at high relative speed or by strong charge separations. These oscillations can lead to rapid exchange of energy and mutual perturbations of orbits among particles.

This in turn will lead to a more uniform spread of energy among particles and a more diffuse and more thoroughly mixed beam profile. The diffusion among particles of energy and the diffusion in space of particle orbits by electrostatic and electromagnetic instabilities can cause changes in the beam profile to some preferred or more stable state. It is these observed instabilities that will be assumed to lead to relaxation. These oscillations can be expected to have frequencies of the order of the beam plasma frequencies or the particle betatron frequencies. The frequencies are characteristic of collective particle motion in the fields of the beam

$$\omega_p^2 = \frac{2eI_x}{\gamma_x M_x a^2 c} \tag{1.8}$$

where M_x is the mass of the particles.

Detailed analysis of the instabilities is very complicated for general beam profiles and will not be necessary for our purposes. We will merely assume that they will exist, will create disorder, and dissipate energy. It will be assumed that the oscillations form waves that will move outward and be lost at a very large radius. The loss of information and energy from the system will be considered irreversible.

Because we are concerned with the evolution of radial profiles we will assume the beam equilibria are initially uniform in the z-direction and that this condition is maintained throughout the equilibrium evolution to a final state. We will thus not allow wave propagation in the z-direction. This wi have the effect of not allowing two stream instabilities to grow from the counterstreaming of the electrons and ions in the z-direction. Because this counterstreaming is assumed to be weak and thus, not a large source of free energy, this should not be a bad approximation.

Certain quantities are always conserved, even in complicated systems, when a system relaxes. Such quantities as total charge per unit length, current in the z-direction and particle number (per unit length) will be assumed to be unchanged by the relaxation. The conservation of current being consistent with our approximation of uniformity and non-propagation of waves in the z-direction.

Therefore, we will assume that collective oscillations on the order of the betatron frequencies of the particles will provide a mechanism for increasing entropy in the beam and removing fee energy. The beam will thus minimize its free energy and maximize its entropy. However, the final state of the beam will have the same magnetic energy per unit length.

1.5   OVERVIEW

In the second chapter of this report the theory of relaxation of plasmoid equilibrium in two-dimensions will be discussed. It will be shown that this relaxation can be studied in all frames through Lorentz invariant energy constraints, that is equivalent to a relaxation in the charge neutral z-pinch frame. Because of this the z-pinch frame will be used for the majority of theory and simulations. In the third chapter, results of simulations will be compared with theory. A final chapter will summarize the results of this research.

SECTION 2
THE THEORY OF RELAXED PLASMOID EQUILIBRIA

2.1 RELAXATION IN THE Z-PINCH FRAME

The simplest place to demonstrate relaxation of a plasmoid to equilibrium is in the z-pinch frame. However, the basic model of plasmoid equilibrium requires Lorentz invariance, so that a complete theory of relaxation should apply in all frames.

One of the key assumptions that one makes in the z-pinch frame to obtain an equilibrium is the assumption of similar density profiles for electrons and ions. In the z-pinch frame this assumption is easy to justify on the grounds that this minimizes the electric field energy, since all charge densities vanish in the z-pinch frame for similar profiles. However, the assumption of similar profiles does not result in the vanishing of electric fields in other Lorentz frames because the charge densities are shifted by differing gamma factors. Therefore, one way to make the Lagrange minimization Lorentz invariant is to find out if similar profiles for electrons and ions are required for an equilibrium or are just a convenient assumption. To do this we will reexamine the variational process in the z-pinch frame for a non-rotating plasmoid and see if anything crucial has been left out.

The variational problem that applied in the z-pinch frame, where electrons and ions were assumed to have similar profiles, could be written:

$$F = \int \frac{B^2}{8\pi} + \lambda n \ln(n) \; 2\pi r dr \;, \tag{2.1}$$

where $B^2/8\pi$ is the magnetic energy and n is the density and $n\ln(n)$ can be considered a thermodynamic entropy. The quantity λ is a Lagrange multiplier (Reference 2). The condition of magnetic energy being minimized while entropy is kept constant yields the condition,

$$\delta F = \delta A \lambda B \Big|_0^\infty + J \delta J A_z + \delta n \ \lambda \ln(n) \ 2\pi r dr \quad , \tag{2.2}$$

where A_z is the vector potential and where we consider $\delta J = e \delta n \ v_z$ and we consider v_z constant.

This yields the equation

$$ev_z A_z + \lambda \ln(n) = 0 \quad , \tag{2.3}$$

and its derivative

$$ev_z n B_\theta = kT \frac{dn}{dr} \quad . \tag{2.4}$$

2.2   THE LORENTZ GENERALIZED VARIATIONAL PRINCIPLE

This last equation is merely the equation for a Bennett pinch (Reference 3) without rotation. The generalized variational principle takes the form

$$F = \frac{E^2 - B^2}{8\pi} + \lambda_1 \ n_i kT_i \ \ln(n_i kT_i) + \lambda_2 \ n_e kT_e \ \ln(n_e \ kT_e) \tag{2.5}$$

$$+ \lambda_3 (n_i^2 - (n_i \beta_i)^2) + \lambda_4 (n_e^2 - (n_e \beta_e)^2) + \lambda_5 (\rho^2 - J^2/c^2) \quad ,$$

where E is the electric field, γ_i and γ_e are the electron and ion gamma factors, respectively, β_i and β_e are the electron and ion velocity normalized to c, and $\rho^2 - J^2/c^2$ is the electromagnetic four current. This generalization means that the Lorentz invariant form of electromagnetic field strength is minimized while Lorentz invariant entropies and densities for both species and the total Lorentz invariant charge density squared are all kept constant. The results of this variation of both n_e and n_i independently, with the usual vanishing of surface terms, can be expressed as:

$$\epsilon(\psi - A_z \rho_i) + \lambda_1 kT_i \ln n_i \, kT_i + \lambda_3 2(n_i - 2\rho_i^2 n_i)$$

$$+ \lambda_5 \, 2\epsilon \, (n_i - n_e - n_i\rho_i^2 - n_e\beta_e\rho_i) = 0 \tag{2.6}$$

and

$$-\epsilon(\psi - A_z \rho_e) + \lambda_2 kT_e \ln n_e \, kT_e + \lambda_4 2(n_e - 2\rho_e^2 n_e)$$

$$+ \lambda_5 \, 2\epsilon \, (n_e - n_i - n_e\rho_e^2 - n_i\rho_e\rho_i) = 0 \tag{2.7}$$

The presence of cross terms in each equation requires $n_i \gamma_e = n_e \gamma_i$ and thus, similar profiles. Similar profiles for electrons and ions, thus, are required by minimization of E^2-B^2 and simultaneous conservation of particle number and total charge. Differentiation leads to the equilibrium equation Equations 1.4a and 1.4b.

2.3   ROTATIONAL EQUILIBRIA AND THE GALACTIC MODE

The inclusion of rotation in the formalism is accomplished by the rotation potential for each specie

$$\Phi_i = m_i \int_0^r \frac{V_{\theta i}^2}{r'} \, dr' \quad , \tag{2.8}$$

$$\Phi_e = m_e \int_0^r \frac{V_{\theta e}^2}{r'} \, dr' \quad . \tag{2.9}$$

where m_i and m_e and $V_{\theta i}$ and $V_{\theta e}$ are ion and electron masses in the frame of interest and rotational velocities respectively. It should be noted that $n_i\Phi_i$ and $n_e\Phi_e$ are Lorentz invariant. Inclusion of strict angular momentum conservation, it should be noted, results in unphysical equilibrium when coupled with nonzero temperature; only uniform density, cosmos filling plasmoids are allowed. However, inclusion of $n_i\Phi_i$ and $n_e\Phi_e$ will result in a general class of rotating equilibria.

The galactic rotation mode results when the condition

$$A_z \propto \Phi_i \propto \Phi_e \quad . \tag{2.10}$$

is applied. This condition occurs when the equilibrium is required to be insensitive to changes in the entropy or temperature of the species. The rotational velocity profile which results in Figure 2 is similar to that seen in galaxies (Reference 5). Some velocity profiles seen in galaxies are shown in Figure 3. The galactic mode velocity profile seen in Figure 2 can be written as,

$$V_\vartheta = \frac{V_o(r/a)}{\sqrt{1+r^2/a^2}} \quad , \tag{2.11}$$

where V_o is an asymptotic equilibrium velocity. The conditions for the derivation of the galactic mode, an insensitivity to temperature, indicate a condition of nearly circular particle orbits. For the more realistic case of very noncircular orbits, the conservation of angular momentum for individual particles will cause a sharp peak to occur in the V_ϑ profile at smaller radii as particles speed up. This can be included by an additional term in Equation 2.11.

$$V_\vartheta = \frac{V_o(1+r/a)}{\sqrt{1+r^2/a^2}} \qquad r>a \quad , \tag{2.12}$$

$$V_\vartheta = \sqrt{2} \, V_o r/a \qquad r<a \quad . \tag{2.13}$$

2.4   AXIALLY MAGNETIZED EQUILIBRIA

The case of rotational motion that produces current, and from this axial magnetization, can be examined by including a Lorentz invariant,

$$j \; \varphi^2 - A^2 \; 2\pi r dr \tag{2.14}$$

while eliminating the invariants associated with entropy. It has been seen that the introduction of axial magnetic flux, of comparable energy to thermal energy, tends to cause a relaxation that is determined by field quantities rather than matter; for this reason thermodynamic entropy seems to be unimportant.
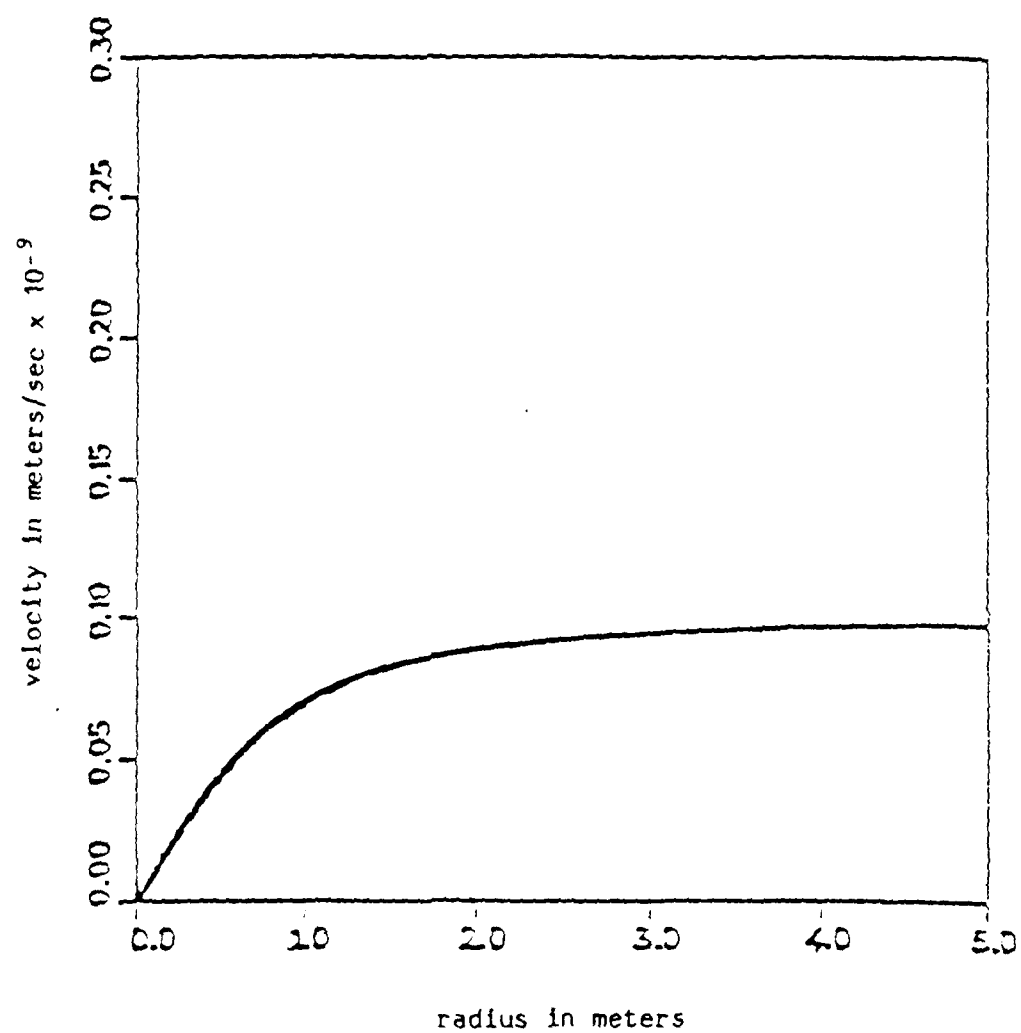
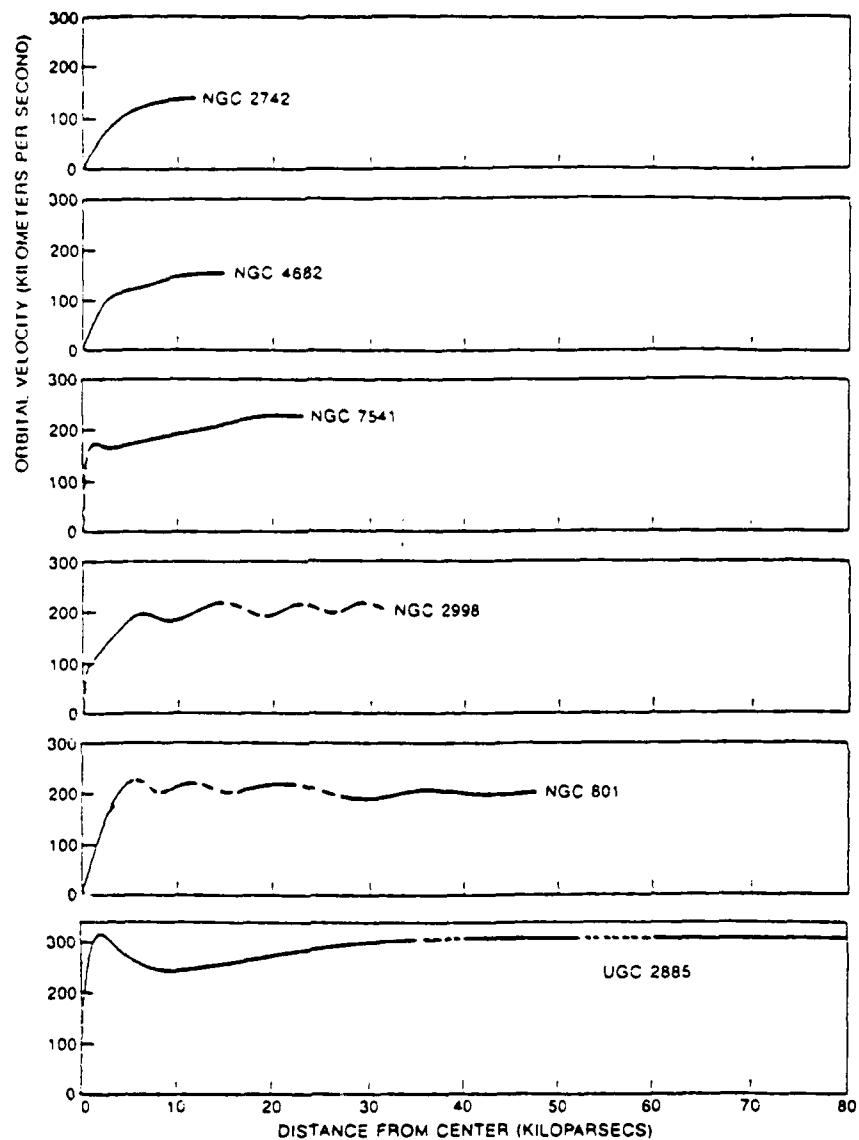Figure 2. A plot of ion rotation velocity versus radius for a plasmoid core similar to that seen in galaxies.

2-5

Figure 3. Rotational curves show orbital velocities of nine S_c galaxies from the center outward. Galaxies increase in luminosity from top to bottom. With increasing luminosity galaxies are larger, orbital velocities are higher and velocity gradients near the galactic center are steeper.

2-6

In the z-pinch frame this reduces to an integral over A^2 only and Lagrange minimization gives the equation for equilibrium (Reference 6).

$$\vec{J} = \lambda \vec{A} \tag{2.15}$$

In this case both axial current and magnetic field assume the form of a Bessel function,

$$J_z \propto B_z \propto J_0\left(\frac{r}{a}\right) \quad . \tag{2.16}$$

SECTION 3

SIMULATIONS OF STEADY-STATE AND RELAXING EQUILIBRIA

3.1   BASIC REQUIREMENTS FOR SIMULATIONS

Simulations of the three types of equilibria discussed were performed. The simulations consisted of equilibria initialized in steady-state and nonequilibrium configurations that were allowed to relax to equilibrium.

The initializing of steady-state fields and currents in two-dimensions in an electromagnetically self-consistent manner proved more difficult than anticipated. However, once numerical integrations over the actual particle charge densities and current were used, instead of idealized analytic ones, simulations could be started without severe electromagnetic turbulence. Because of the fully electromagnetic nature of the code MAGIC (Reference 7), the limitation on the timestep was determined by the Courant condition

$$\Delta t > \sqrt{2}\, \frac{\lambda}{c} \quad , \tag{3.1}$$

where λ is the dimension of the smallest cell. This required thermal speeds for ions and electrons that were close to the speed of light

$$V_{th} = c/3 \quad . \tag{3.2}$$

This allowed significant particle dynamics to occur within a reasonable number of timesteps.

3.2   SIMULATIONS OF NONROTATING EQUILIBRIA

Simulations were performed in the z-pinch frame for reasons of simplicity. Simulations of nonrotating equilibria were performed first. The Bennett profile was observed to be stable for many thermal transit times:

$$\tau = \frac{a}{V_{th}} \quad , \tag{3.3}$$

where a is the Bennett radius. In Gaussian units, this is written:

$$a^2 = \frac{mV_{th}^2}{2\pi n_0 e^2 p^2} \quad , \qquad\qquad (3.4)$$

where m is the ion or electron mass, n_0 is the central number density, and p is the drift velocity normalized to the speed of light. A level of fluctuations of about 10% of the mean magnetic field and particle density was observed even in steady-state, as can be seen in Figure 4. Electrostatic oscillations were observed to be strongest in the center of the pinch.

A nonequilibrium, nonrotating beam was also simulated and allowed to relax to equilibrium. The equilibrium appeared to approach the Bennett state after many thermal transit times. Density profiles of the initial and final states are shown in Figure 5. This period of diffusion was much longer than expected. The expected time for significant diffusion to occur was a few thermal transit times.

3.3   ROTATING, NON AXIALLY MAGNETIZED EQUILIBRIA

The case of rotating equilibria was simulated by being initialized in both steady-state and nonequilibrium configurations. A steady-state rotational velocity profile, consistent with a Bennett-like profile, was arrived at by including the effect of large radial excursions of particles. This modified the originally predicted galactic mode as is shown in Figure 6. The rotating nonequilibrium plasmoid was observed to relax to a rotation and density profile close to the previously simulated steady-state.

3.4   AXIALLY MAGNETIZED EQUILIBRIA

The case of rotation with axial magnetization was simulated in both steady-state and nonsteady-state cases. An approximately Bennett- or Bessel-function-shaped profile with a modified galactic mode was observed for
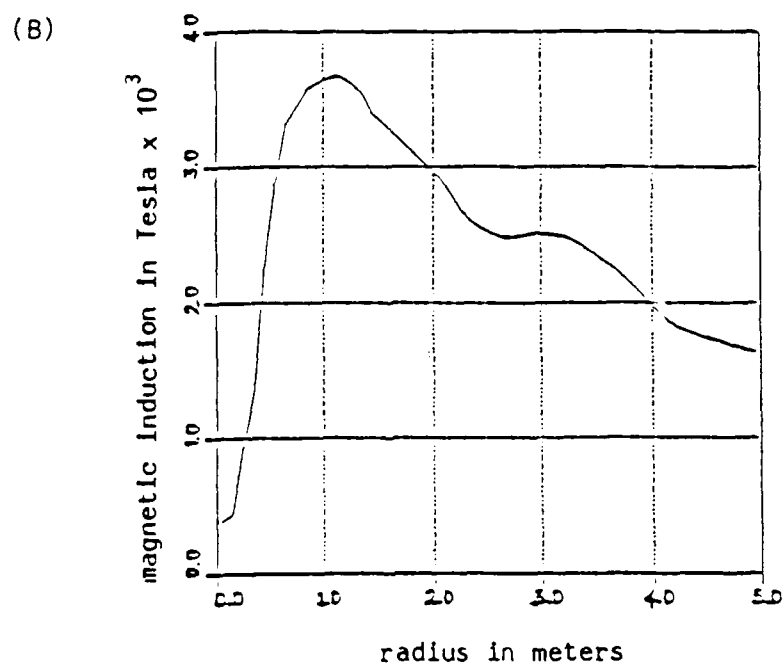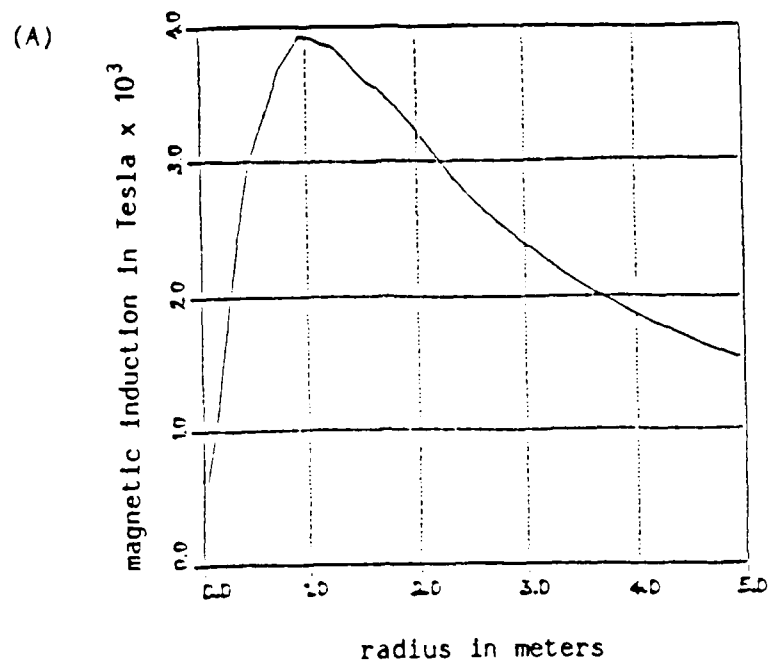
(A)

radius in meters

(B)

radius in meters

Figure 4. Initial (A) and final (B) magnetic field profiles for a Bennett
plasmoid. Note large fluctuations visible in final state. Such
fluctuations occurred throughout the simulation and appeared to
be a feature of the equilibrium.

3-3

(A)

current density Amperes/(meters)^2 x 10^{-3}

radius in meters

(B)

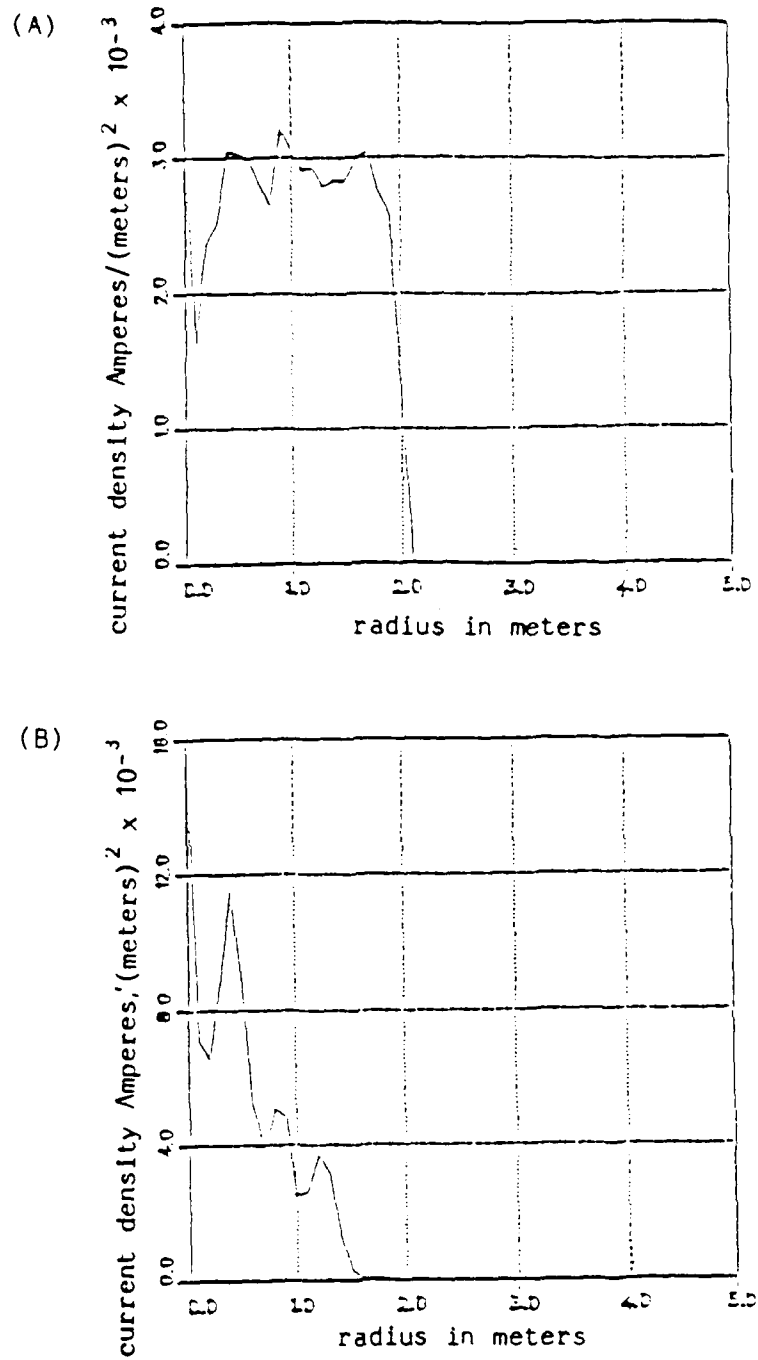current density Amperes,(meters)^2 x 10^{-3}

radius in meters

Figure 5. Initial (A) and final (B) density profiles for a plasmoid core
equilibrium. Note the presence of strong density fluctuations in
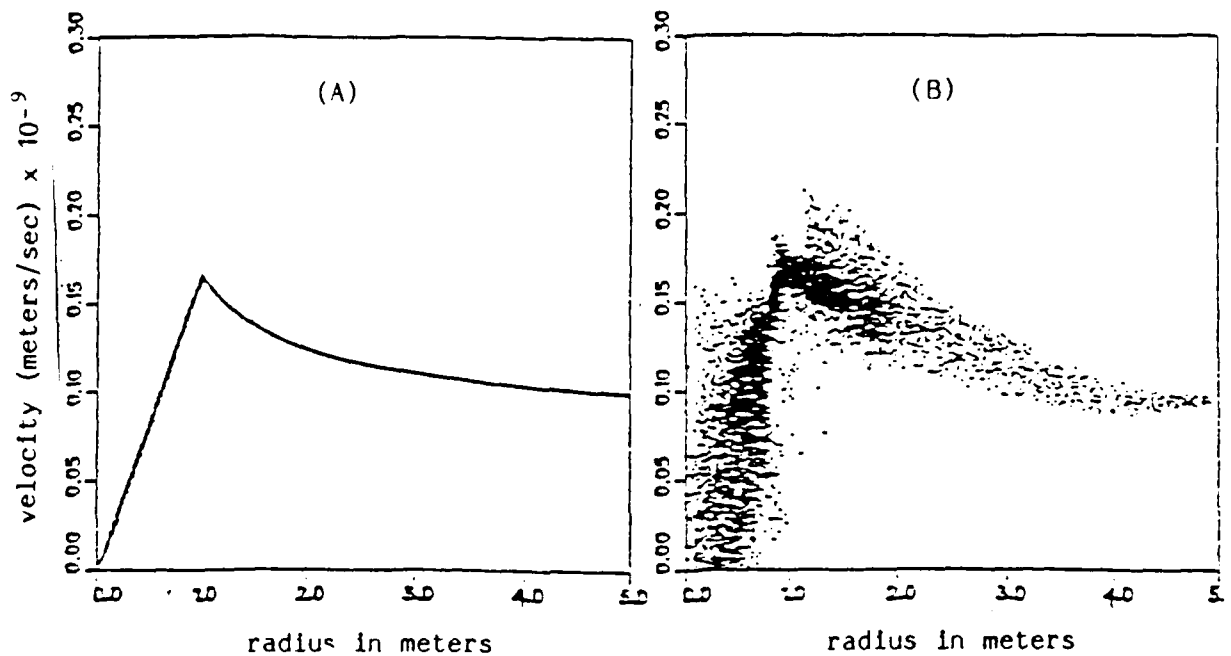both initial and final states.

3-4

Figure 6. θ- velocity versus radius from MAGIC simulations at (A) initial time, and (B) after ten particle transit times, for an equilibrium plasmoid. The "galactic" profile has been preserved with only a small amount of diffusion.

the steady-state rotation profiles in both cases. As shown in Figure 7, the initial v and axial magnetic field profiles for an approximate steady-state profile were preserved after many particle transit times with only minor changes. The final state of a nonsteady-state initial profile also appeared to relax to this state, though the initially nonsteady-state profile was observed to have regions of axial magnetic field reversal in late times. In general, the axially magnetized state took longer to relax than those states without axial magnetizaton.

3.5   RELAXATION TIMES

Diffusion from uniform profiles to Bennett or Bessel function steady-state profiles took much longer than the predicted thermal transit time. Instead, the diffusion timescale appeared consistent with a Bohm diffusion time (Reference 5):

$$\tau_B = 16 \frac{d}{c\beta} . \qquad\qquad (3.5)$$

This is on the order of 100 thermal transit times. To test this a simulation was run with the same Bennett radius, but with higher drift velocity, thus giving a shorter Bohm time. Signs of enhanced diffusion were observed; however, much longer simulations would have to be run to give definitive proof that the Bohm time in fact controls the diffusion process.
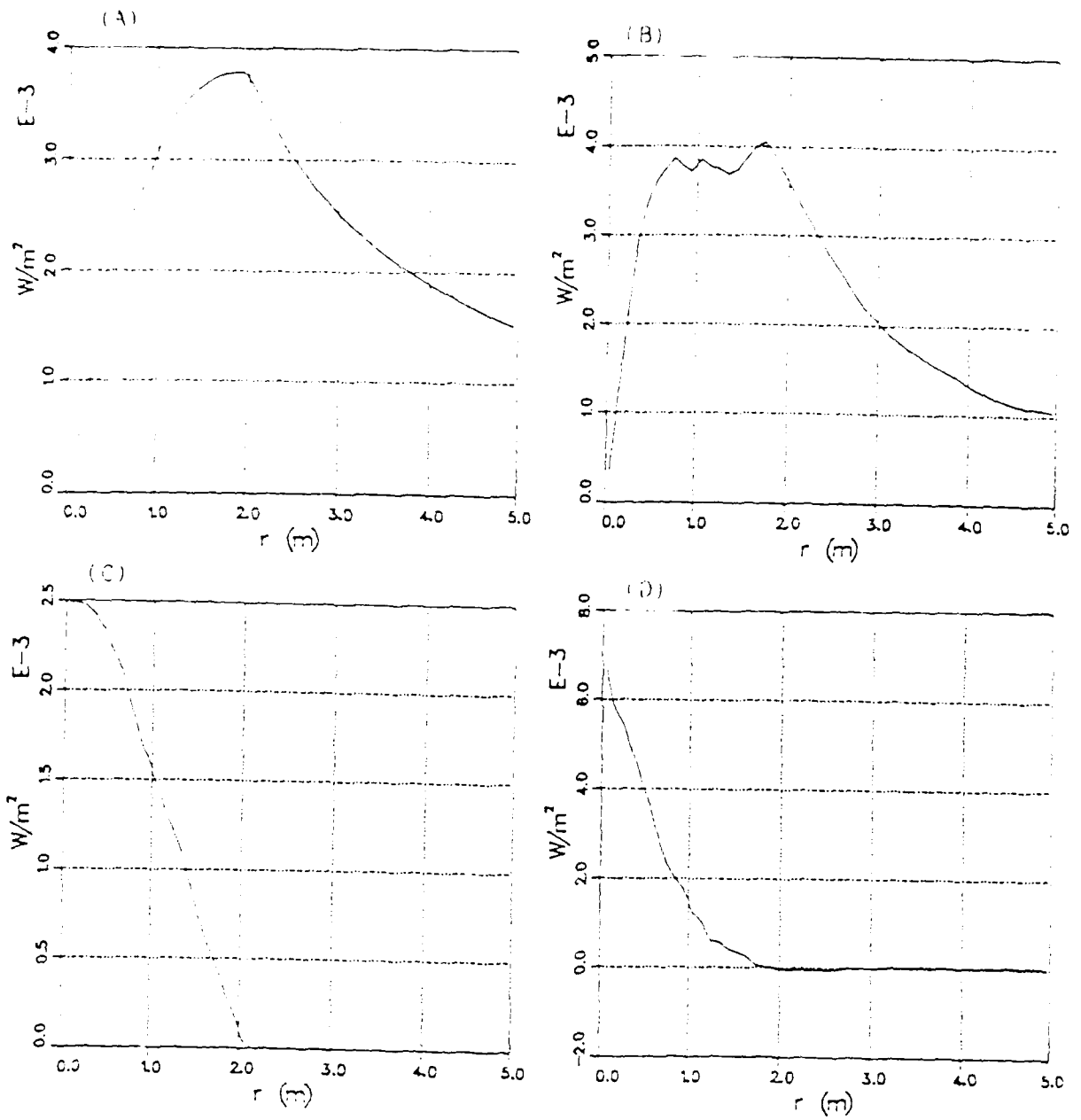
Figure 7. Initial (A) and final (B) θ magnetic field profiles for an axially magnetized plasmoid core equilibrium. Axial magnetic field profiles for initial (C) and final (D) states of the same equilibrium.

SECTION 4
SUMMARY AND FUTURE WORK

The program of theory and simulations has made great progress. Steady-state equilibria of three basic types have been identified theoretically and verified with simulations. The theory that a Lorentz invariant variational problem could predict relaxed, robust plasmoid core equilibria has been verified. In addition, the relaxation process has been observed to occur for these three cases and its timescale has been approximately characterized. This progress can serve as a foundation for an expanded effort to define plasmoids in three dimensions as well as address the problem of propagation of plasmoids across magnetic fields.

SECTION 5

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